Change When the Sun Rises

by Olen Rambow

We wish to find the amount of time by which sunrise is delayed when you make one full trip around the Earth going eastward at the equator with speed v.

Let M, R, and ω_0 be the mass, radius, and angular velocity of the Earth, respectively, and let m be your mass. Approximating the Earth as a solid ball of uniform density and assuming that both you and the Earth are rotating about the center of the Earth (i.e., ignoring the tiny effect that you have on the location of the center of mass of the system), the total angular momentum of the whole system is then

$$L = \tfrac{2}{5}MR^2\omega_0 + mR^2\omega_0$$

After you push off from the Earth's surface and start walking at a constant speed v, the Earth rotates at some reduced angular velocity that we'll call ω_1. Your linear speed is then $\omega_1 R + v$, so the total angular momentum of the system can now be expressed as

$$L = \left[\tfrac{2}{5}MR^2\omega_1\right] + [mR(\omega_1 R + v)]$$

where the first term in brackets is the reduced angular momentum of the Earth and the second term is your angular momentum. Setting the above two equations equal and solving for the difference between ω_0 and ω_1, we get

$$\Delta\omega = \omega_0 - \omega_1 = \frac{v}{\left(1 + \frac{2M}{5m}\right)R}$$

This is the amount by which the Earth's angular velocity is reduced during the time you're traveling at speed v. The amount of angular rotation $\Delta\theta$ by which the Earth is delayed is then $\Delta\omega$ multiplied by the amount of time it takes you to complete your trip, which is $\frac{2\pi R}{v}$. Thus,

$$\Delta\theta = \frac{2\pi}{1 + \frac{2M}{5m}}$$

What we're interested in is the amount of time it takes the Earth to rotate through this angle, because that's the amount of time by which the sunrise will be delayed. This is given by

$$\Delta t = \frac{\Delta\theta}{\omega_0}$$

The Earth's angular velocity, ω_0, is just 2π radians divided by the number of seconds in a day, and the Earth's mass is 5.972×10^{24} kg. Plugging these things in, we get

$$\Delta t = 2.17 \times 10^{-18}\ \mathrm{s}$$

That's the amount of time by which you will delay all future sunrises if you travel around the world at the equator. (It's a very tiny amount.) To find how much mass you would need to delay the sunrise by one full second, simply set Δt equal to one and solve the previous equation for m.