Special Relativity

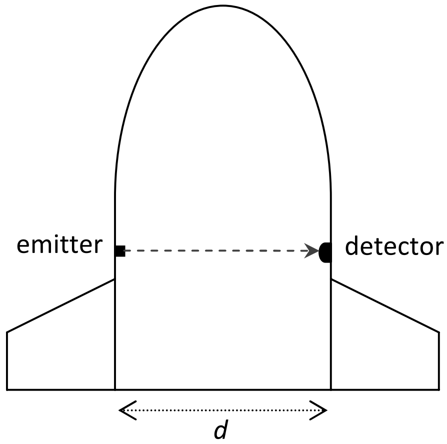
by Olen Rambow

At some unfortunate point in history, scientists discovered that the speed of light (when it is traveling through free space) is constant, regardless of how you're moving when you measure it. If you're sitting still and a beam of light shoots past you, the light will appear to be moving at a speed of 3×10^8 m/s; and if you're moving alongside the beam of light at some really high speed, say 2.9×10^8 m/s, the light will *still* appear to be moving 3×10^8 m/s faster than you. This sounds impossible, even stupid, but it has been verified countless times by experiment. The discovery that the speed of light (in a vacuum) is always constant has a bunch of weird consequences that have really screwed up the field of physics. Below is a derivation of one such consequence called "time dilation."

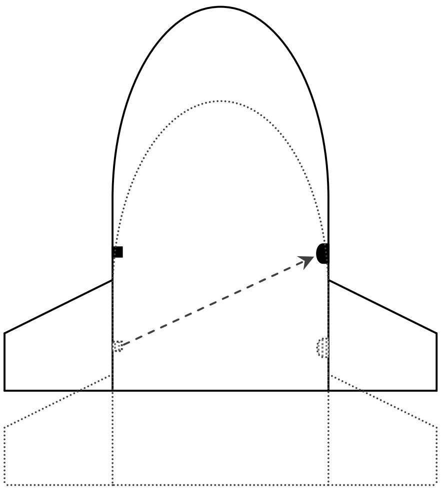
Consider the following situation. A spaceship traveling at speed v has a photon emitter mounted on one wall and a photon detector mounted on the opposite wall. The width of the spaceship, and thus the distance from the emitter to the detector, is d. There are two observers, one inside the ship and one outside. A single photon is fired from the emitter and goes directly to the detector, and both observers measure the distance traveled by the photon and the amount of time that it takes the photon to travel from the emitter to the detector.

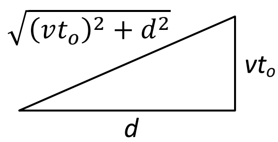
On the right is a diagram showing what the observer inside the ship sees. The photon travels a distance d in time t_i, and since photons always travel at speed c, we know that

$$\frac{d}{t_i} = c$$



Now consider the observer outside of the ship. He measures the time that it takes the photon to travel from the emitter to the detector to be t_o. (Ordinarily, we would expect t_o to be the same as t_i, but it will turn out that since c is the same for both observers, t_i and t_o *must* be different.) During the photon's flight, the spaceship travels forward a distance vt_o. The detector and photon both travel forward along with the ship a distance of vt_o, and therefore the photon travels a little bit farther as seen from outside of the ship. The distance traveled by the photon is the hypotenuse of a right triangle with legs d and vt_o.





The speed of the photon as measured by the outside observer is $\frac{\sqrt{(vt_o)^2+d^2}}{t_o}$, which must also equal *c*. We then have that the speed of the photon as measured by the inside observer equals the speed as measured by the outside observer:

$$\frac{d}{t_i} = \frac{\sqrt{(vt_o)^2 + d^2}}{t_o}$$

Note that the numerator of the fraction on the right is larger than the numerator on the left. In order for the fractions to be equal, the denominator on the right must also be larger than the denominator on the left. ***This means that the amount of time that passed outside of the ship must be greater than the amount of time that passed inside of the ship.*** Meditate on this for a while.

To find the relationship between t_i and t_o, we just need to do some simple algebra to eliminate *d*. From $\frac{d}{t_i} = c$, we get that $d = ct_i$. Substitute this expression for *d* into the equation $c = \frac{\sqrt{(vt_o)^2+d^2}}{t_o}$, and we get

$$c = \frac{\sqrt{(vt_o)^2+(ct_i)^2}}{t_o}.$$

Multiplying both sides by t_o and squaring both sides, we get

$$(ct_o)^2 = (vt_o)^2 + (ct_i)^2$$

Moving both terms containing t_o to one side and factoring out t_o, we get

$${t_o}^2(c^2 - v^2) = c^2{t_i}^2$$

This can be rearranged to yield

$$t_i = \left(\sqrt{1 - \frac{v^2}{c^2}}\right)t_o$$

Physicists generally write this as $t_o = \left(\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}\right)t_i$, and they abbreviate $\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$ with the symbol γ (the Greek letter gamma) and call it the "Lorentz factor."

Note: Mr. Rambow typed this up quickly while proctoring an exam, and it is quite likely riddled with errors. If you find any, don't be alarmed.